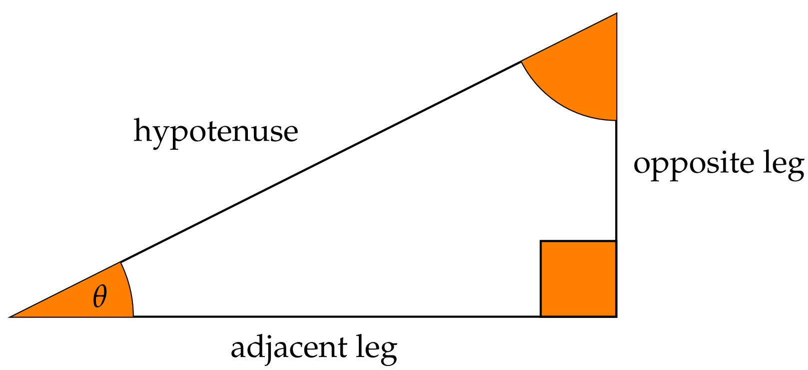
Purpose: In this problem set, you will explore the trigonometry of right triangles using the sine, cosine, and tangent functions. This is seemingly unrelated to the trigonometry we used on circles, but we will connect these two soon.

So far, we've only looked at two trigonometric functions and we investigated them in the context of circles. Now, we're going to look at those functions with any old right triangle but keep in mind that these triangles all live as reference triangles inside circles (but more about that later).

For today, we're going to live in the world of triangles.

In case you don't recall, we need a way to distinguish the legs of a right triangle from a fixed angle, θ (spelled "theta"). The diagram below will help us keep everything in order.

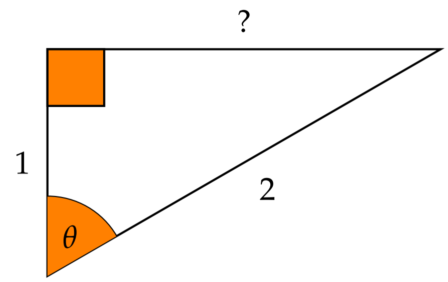


1. Draw three right triangles spun around in three ways (upside down, tilt-y, backwards, whatever). Pick one of the non-right angles and label the opposite leg, the adjacent leg, and the hypotenuse.

Just like with measuring ratios in circles, we'll measure ratios based on triangles too. Using just one triangle, we can come up with *six* ratios! Today, we'll focus on three.

$$\frac{\text{opposite}}{\text{hypotenuse}}, \quad \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \frac{\text{opposite}}{\text{adjacent}}$$

2. Consider the triangle below. The value of θ is 60°.



   (a) What is the length of the side opposite to θ? Adjacent to θ? The hypotenuse?

   (b) Compute (either exactly or using a calculator) each ratio below:

$$\frac{\text{opposite}}{\text{hypotenuse}} = \qquad \frac{\text{adjacent}}{\text{hypotenuse}} = \qquad \frac{\text{opposite}}{\text{adjacent}} =$$

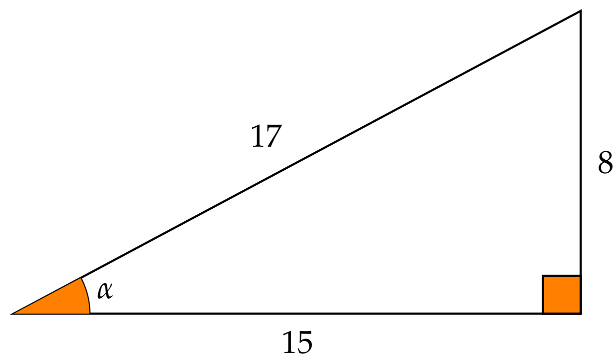
   (c) Using a calculator, compute the three trigonometric functions below. The last one is called *tangent* and we haven't defined it yet, but we will! (Notice that the angle is given in degrees and not radians.)

$$\sin(\theta) = \qquad \cos(\theta) = \qquad \tan(\theta) =$$

   (d) What are your observations? How does this connect to circles?

3. Draw another right triangle and specify one of the angles to be θ so that all of the trig functions precisely match those of the triangle in Question 2. How do you know?

4. Consider the triangle below.



   (a) Compute $\sin(\alpha)$.

   (b) Compute $\cos(\alpha)$.

   (c) Compute $\tan(\alpha)$.

   (d) Label the other non-right angle β. Compute sine, cosine, and tangent of β.

   (e) Compare the computations of the trigonometric functions for the two angles.

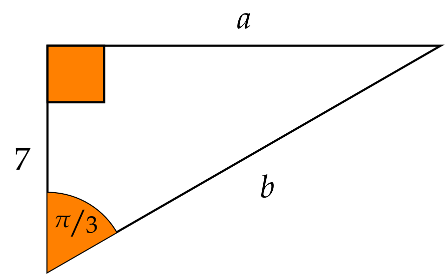
Important Identities: Your last observation may lead you to guess the following identities for sine and cosine, called the **co-function identities**:

$$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right), \qquad \cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right),$$

where θ is measured in radians.

For the rest of this problem set, we're going to solve some puzzles.

5. Consider the triangle below. Our goal is to fill in the missing lengths of the legs.



   (a) Write $\cos\left(\frac{\pi}{3}\right)$ using this triangle. (You'll have an unknown.)

   (b) Compute $\cos\left(\frac{\pi}{3}\right)$ using the notes from Friday, a calculator, or other means. (This should be a number.)

   (c) Using (a) and (b), solve for one unknown.

   (d) Now you're ready to find the other unknown using a famous theorem! Do so now.

6. A right triangle has one angle of $\frac{\pi}{6}$ and a hypotenuse of 20. Find the unknown sides and angles of the triangle.

7. A right triangle has one angle of 30° and the leg opposite to that angle is 7 units long. Find the missing side lengths and angles. (Careful to set up the correct picture here.)

8. To find the height of a tree, a person walks to a point 20 feet from the base of the tree, and measures the angle from the ground to the top of the tree to be 61 degrees. Find the height of the tree.